Exercise 30

Find the limit or show that it does not exist.

$$\lim_{x\to\infty} \sqrt{x^2+1}$$

Solution

Factor the highest power of x and then use the limit laws.

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{x^2+1} &= \lim_{x\to\infty} \sqrt{x^2\left(1+\frac{1}{x^2}\right)} \\
&= \lim_{x\to\infty} \left(x\sqrt{1+\frac{1}{x^2}}\right) \\
&= \left(\lim_{x\to\infty} x\right)\left(\lim_{x\to\infty} \sqrt{1+\frac{1}{x^2}}\right) \\
&= \left(\lim_{x\to\infty} x\right)\left[\sqrt{\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)}\right] \\
&= \left(\lim_{x\to\infty} x\right)\left(\sqrt{\lim_{x\to\infty} 1+\lim_{x\to\infty}\frac{1}{x^2}}\right) \\
&= \left(\lim_{x\to\infty} x\right)(\sqrt{1+0}) \\
&= \lim_{x\to\infty} x \\
&= \infty
\end{aligned}$$